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Stabilisation of Seven Directions in an Early Universe M – theory Model

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ABSTRACT

Our model consists of intersecting $22'55'$ branes in M theory distributed uniformly in the common transverse space. Equations of state follow from U duality symmetries. In this model, three spatial directions expand, and seven directions stabilise to constant sizes. From string theory perspective, the dilaton is hence stabilised. The constant sizes depend on certain imbalance among initial values. One naturally obtains $M_{11} \simeq M_s \simeq M_4$ and $g_s \simeq 1$ within a few orders of magnitude. Smaller numbers, for example $M_s \simeq 10^{-16} \; M_4$, are also possible but require fine tuning.

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1. In early universe, the temperature and energy densities are high. When they are of the order of Planck scale $M_4 \simeq 10^{19} GeV$, the dynamics of the early universe is expected to be described by a more fundamental theory such as string theory or M theory [1, 2].

If this is the case then the problem of spacetime dimensions needs to be resolved – spacetime is eleven dimensional in M theory whereas it is four dimensional in our observed universe.

A canonical resolution is that the early universe starts out being eleven dimensional. During its evolution, by some dynamics, seven of the spatial directions cease to expand and their sizes become stabilised. The remaining three spatial directions continue to expand and become the observed universe.

The stabilised sizes then relate the M theory scale M_{11} and the four dimensional Planck scale M_4 . Likewise, since string theory can be obtained by dimensionally reducing M theory, the sizes also relate M_{11} and the string scale M_s and the string coupling constant g_s . One may then enquire, for example, whether it is possible to have string/M theory scale in the TeV range as required in Large Volume compactification scenarios [3].

Various proposals have been made for obtaining a four dimensional universe from string/M theory [4, 5, 6]. Typically, one assumes that the spatial directions are all toroidal, and are wrapped by a gas of winding and anti winding strings or p–branes; and that the cosmological evolution is governed by a ten/eleven dimensional effective action. The earliest proposal [4], in the context of string theory, is based on the observation that winding and anti winding strings oppose the expansion, and are annihilated efficiently in four dimensional spacetime. Others [5, 6] are variants of this, or based on its generalisations to winding and anti winding p–branes in string/M theory. These proposals are quite appealing and have been used in a variety of 'brane gas' models [5, 6], but some important issues yet remain to be resolved [7].

In this letter, based on the ideas in [8, 9], we present an M theoretic early universe model where seven of the spatial directions cease to expand and their sizes become stabilised. From string theory perspective, the dilaton is hence stabilised. The remaining three spatial directions continue to expand, thus leading to a four dimensional universe. The stabilised sizes, and thus the explicit relations among $(M_{11}, \; M_4, \; M_s, \; g_s)$, depend on certain imbalance among initial values. The exact values are obtained numerically, but can also be estimated analytically under certain approximations. In this model, one may obtain any value for M_{11} or M_s , including in the TeV range, by a

corresponding fine tuning of initial values.

2. Our model is as follows. Let all the spatial directions be toroidal. Consider mutually BPS intersecting brane configurations in M theory where N sets of coincident branes and antibranes intersect as per the rules given in [10]. According to these rules, for example, two sets of 2 branes must intersect along zero common direction, 2 branes and 5 branes along one common direction, or two sets of 5 branes along three common directions.

The branes and antibranes in such a configuration differ significantly from those in brane gas models, as explained in section 2.6 of the first and section 6 of the second paper in [8]. Briefly, the differences are the following: (1) In brane gas models, the branes can intersect each other arbitrarily. Here the intersections must follow specific rules. U duality symmetries of M theory then imply a relation among the equations of state which turns out to be a crucial element underlying the present results [9]. (2) The branes in brane gas models support excitations on their surfaces and, at high energies, have $S \sim \mathcal{E}$ where S is the entropy and $\mathcal{E}$ the energy. Here, the intersecting branes form bound states, become fractional, support very low energy excitations and, hence, are highly entropic. At high energies, $S \sim \mathcal{E}^{\frac{N}{2}}$ which, for $N > 2$, vastly exceeds the entropy in brane gas models. Such intersecting brane configurations are, therefore, the entropically favourable ones. (3) In brane gas models, the branes tend to annihilate if they intersect each other. Here, the intersections are necessary for formation of bound states and of high entropic excitations. These excitations are long–lived and non interacting to the leading order, hence the branes here are metastable and do not immediately annihilate. See [8] for more details, and [11] also.

In our model, we consider $N = 4$ intersecting brane configuration denoted by $22'55'$, which has vanishing net charges and consists of two sets each of 2 branes and 5 branes along (x^1, x^2) , (x^3, x^4) , $(x^1, x^3, x^5, x^6, x^7)$, and $(x^2, x^4, x^5, x^6, x^7)$ directions. This configuration, when localised in the common transverse space along (x^8, x^9, x^{10}) directions, describes a four charged black hole [12]. Here, we take the configuration to be uniformly distributed in the common transverse space which then is assumed, as in [8, 9], to describe a homogeneous anisotropic universe whose evolution is governed by an eleven dimensional effective action.

Let $I = 1, 2, 3, 4$ denote the branes $2, 2', 5, 5'$ respectively. We assume

that, as in the case of black holes, the energy momentum tensors $T^A_{\ B(I)}$ of the I^{th} set of branes are mutually non interacting and seperately conserved [8, 11]. Then

$$T^A_{\ B} = \sum_I T^A_{\ B(I)} \ , \quad \sum_A \nabla_A \, T^A_{\ B(I)} = 0 \tag{1}$$

where $T^A_{\ B}$ is the total energy momentum tensor of the configuration. Homogeneity implies that $T^A_{\ B} = diag \ (-\rho, p_i)$ and $T^A_{\ B(I)} = diag \ (-\rho_I, p_{iI})$. We take $\rho_I > 0$.

To obtain the equations of state $p_{iI}(\rho_I)$, let $p_{\parallel I}$ and $p_{\perp I}$ denote parallel and perpendicular components of pressure due to I^{th} set of branes. For the mutually BPS intersecting brane configurations of the type considered here, it is shown in [9] that U duality symmetries of M theory imply that the functions $p_{\perp I}(\rho_I)$ must be same for all I and that $p_{\parallel I} = 2p_{\perp I} - \rho_I$. For the $22'55'$ configuration, it then folows that if ρ_I are all equal then, for any function $p_\perp(\rho)$, the seven brane directions become stabilised and the remaining three spatial directions expand [9].

However, an explicit form for the function $p_\perp(\rho)$ is required to obtain further details such as the values of the stabilised sizes, or to understand the evolution when ρ_I are all not equal. In principle, $p_\perp(\rho)$ is to be determined by brane antibrane dynamics. But not much is known about this dynamics. Hence, in order to make progress and to understand the details of the evolution, we assume in our model that $p_\perp = (1 - u) \ \rho$ where u is a constant. Such a form, with $u = 1$, is indeed derived in [8] in the limit where the brane antibrane annihilation can be neglected. Here, we will keep u an arbitrary constant, assuming only that $0 < u < 2$. The resulting evolution is then applicable, atleast qualitatively, even if u is varying *e.g.* due to brane antibrane annihilation effects.

It then follows that $p_{iI} = (1 - u^I_i) \ \rho_I$ where, for the $22'55'$ configuration,

$$\begin{aligned} u^1_i &= u \ (2,2,1,1,1,1,1,1,1,1) \\ u^2_i &= u \ (1,1,2,2,1,1,1,1,1,1) \\ u^3_i &= u \ (2,1,2,1,2,2,2,1,1,1) \\ u^4_i &= u \ (1,2,1,2,2,2,2,1,1,1) \ . \end{aligned} \tag{2}$$

3. Consider now the evolution of the $D = (10 + 1)$ – dimensional homogeneous anisotropic universe in the model described above. Let the line

element ds , with $x^A = (t,\ x^i)$ and $i = 1, 2, \cdots, D-1$, be given by

$$ds^2 = \sum_{AB} g_{AB} dx^A dx^B = -dt^2 + \sum_i e^{2\lambda^i}\ (dx^i)^2 \tag{3}$$

where λ^i are functions of t only. Einstein equations $R_{AB} - \frac{1}{2} g_{AB} R = T_{AB}$, with $8\pi G = 1$, and equations (1) lead to $\rho_I = e^{l^I - 2\Lambda}$ and

$$\sum_{ij} G_{ij} \lambda^i_t \lambda^j_t = 2 \sum_I e^{l^I - 2\Lambda} \tag{4}$$

$$\lambda^i_{tt} + \Lambda_t \lambda^i_t = \sum_I u^{iI}\ e^{l^I - 2\Lambda} \tag{5}$$

where $l^I = \sum_i u^I_i \lambda^i + l^I_0$, $\Lambda = \sum_i \lambda^i$, the subscripts t denote time derivatives, and

$$G_{ij} = 1 - \delta_{ij} \quad , \quad G^{ij} = \frac{1}{D-2} - \delta^{ij} \quad , \quad u^{iI} = \sum_j G^{ij} u^I_j \quad . \tag{6}$$

Let $d\tau = e^{-\Lambda}\ dt$ and $\mathcal{G}^{IJ} = \sum_i u^{iI} u^J_i$. Also, define $\mathcal{G}_{IJ}$ by $\sum_J \mathcal{G}^{IJ}\ \mathcal{G}_{JK} = \delta^I_K$. Then, manipulating equations (4) and (5), one obtains

$$\lambda^i = \sum_{IJ} \mathcal{G}_{IJ}\ u^{iI}\ (l^J - l^J_0) + L^i\ \tau \tag{7}$$

$$l^I_{\tau\tau} = \sum_J \mathcal{G}^{IJ}\ e^{l^J} \tag{8}$$

$$\sum_{IJ} \mathcal{G}_{IJ}\ l^I_\tau l^J_\tau = 2\ (E + \sum_I e^{l^I}) \tag{9}$$

where the subscripts τ denote τ–derivatives, L^i are integration constants satisfying $\sum_i u^I_i L^i = 0$, and $2\ E = - \sum_{ij} G_{ij} L^i L^j$. Also, with no loss of generality, we have taken the initial values to be

$$\Big(\lambda^i,\ \lambda^i_t,\ l^I,\ l^I_t,\ \rho_I,\ \tau\Big)_{t=0} = \Big(0,\ k^i,\ l^I_0,\ K^I,\ \rho_{I0},\ 0\Big) \tag{10}$$

where $\rho_{I0} = e^{l^I_0}$ and $k^i = \sum_{IJ} \mathcal{G}_{IJ}\ u^{iI}\ K^J + L^i$. For the $22'55'$ configuration in our model, u^I_i are given in equations (2) using which u^{iI} , $\mathcal{G}^{IJ}$, and $\mathcal{G}_{IJ}$ can be calculated easily. For example,

$$\mathcal{G}^{IJ} = 2u^2\ \Big(1 - \delta^{IJ}\Big) \quad , \qquad \mathcal{G}_{IJ} = \frac{1}{6u^2}\ (1 - 3\delta_{IJ}) \quad . \tag{11}$$

We now point out an interesting similarity with black holes: When L^i all vanish, e^{λ^i} here have the same form as those for extremal $22'55'$ black holes and e^{2uh_I} , where $h_I = \sum_J \mathcal{G}_{IJ} \ (l^J - l_0^J)$, play the role of harmonic functions $H_I = 1 + \frac{Q_I}{r}$. Compare equations (7) here and (18) in [12]. Also, the asymptotic limit $t \to \infty$ here, see below, corresponds to the near horizon limit $r \to 0$ and (certain combination of) ρ_{I0} play the role of Q_I .

4. To obtain $\lambda^i(t)$ for the $22'55'$ configuration, and thus the evolution of the universe, one may solve equations (8) – (11) for $l^I(\tau)$ and obtain $\lambda^i(\tau)$ from equation (7). Then $t(\tau)$ and, hence, $\tau(t)$ follow from $dt = e^{\Lambda} \ d\tau$. We are unable to solve equations (8) – (11) analytically. Nevertheless, the important features of the evolution can be obtained as follows.

For the $22'55'$ configuration, the following two results can be proved: **(R1)** The constraints $\sum_i u_i^I L^i = 0$ imply that $0 \le c_i \ (L^i)^2 \le E$ where c_i are constants of $\mathcal{O}(1)$. Hence $E = 0$ if and only if $L^i = 0$ for all i . **(R2)** If $E \ge 0$ then equations (7) and (9) imply that none of $(\Lambda_\tau, \ l_\tau^I)$ may vanish, and that they must be *all positive* or *all negative*.

Let $K^I = l_\tau^I(0) > 0$ for all I . The above results together with equations (8) and (11) then imply that, as τ increases, $l^I(\tau)$ all increase and diverge at finite $\tau = \tau_\infty$. In the limit $\tau \to \tau_\infty$ and to the leading order, we obtain

$$e^{l^I} = \frac{1}{3u^2} \, \frac{1}{(\tau_\infty - \tau)^2} \quad , \quad t = t_* + A \ (\tau_\infty - \tau)^{- \frac{2-u}{u}}$$

$$e^{\lambda^i} = e^{v^i} \left(\frac{1}{3u^2} \ \frac{1}{(\tau_\infty - \tau)^2} \right)^{\sum_{IJ} \mathcal{G}_{IJ} \, u^{iJ}} = e^{v^i} \ \left\{ B \ (t - t_*) \ \right\}^{\beta^i} \qquad (12)$$

where t_* and τ_∞ are finite constants and depend on the details of evolution, A and B are u–dependent constants, $v^i = - \sum_{IJ} \mathcal{G}_{IJ} \ u^{iI} \ l_0^J + L^i \ \tau_\infty$, and $\beta^i = \frac{2u}{2-u} \ \sum_{IJ} \mathcal{G}_{IJ} \ u^{iJ}$. Explicitly, β^i are given by

$$\beta^i = \frac{2}{3(2-u)} \, (0, \ 0, \ 0, \ 0, \ 0, \ 0, \ 0, \ 1, \ 1, \ 1) \ . \qquad (13)$$

Thus, asymptotically, $t \to \infty$ since $0 < u < 2$ in our model. And, $e^{\lambda^i} \to t^{\frac{2}{3(2-u)}}$ for the common transverse directions $i = 8, 9, 10$. Hence, these directions continue to expand, their expansion being precisely that of a $(3+1)$ – dimensional homogeneous, isotropic universe containing a perfect

fluid whose equation of state is $p = (1-u)\,\rho$. Also, $e^{\lambda^i} \to e^{v^i}$ for the brane directions $i = 1, \cdots, 7$. Hence, these directions cease to expand and their final sizes are given by e^{v^i} .

In our model, irrespective of initial values, three spatial directions will always expand and seven brane directions will always be stabilised and reach constant sizes. The underlying dynamics is distinct from those in [4, 5, 6] and can be described as follows. It follows from equation (5) that parallel brane directions contract and transverse ones expand, at opposite rates for 2 branes and 5 branes. If the brane energy densities ρ_I are all different then, generically, so will be the corresponding expansion and contraction rates, and the brane directions will have net expansion or contraction. Only if the expansion rates equal contraction rates, will the brane directions cease to expand or contract and their sizes stabilise to constant values.

Such an equality ensues eventually in our model as a result of two crucial features : **(i)** The dynamics of the evolution, given by u_i^I which in turn follow from U duality symmetries [9], is such that ρ_I , even if different initially, evolve to become all equal. This equality is due to each $\rho_I \sim e^{l^I}$ being 'sourced' by the sum of other three, see equations (8) and (11). **(ii)** The $22'55'$ configuration is such that each brane direction is parallel to two sets of branes, and transverse to other two in just the right way. Hence, its expansion and contraction rates become equal once ρ_I become all equal.

The stabilised sizes of the brane directions should then depend on the imbalance among ρ_{I0} and $\lambda_t^i(0)$. Indeed we have, for example,

$$e^{v^1} = e^{L^1\,\tau_\infty} \quad \left(\frac{\rho_{20}\;\rho_{40}^2}{\rho_{30}\;\rho_{10}^2} \right)^{\frac{1}{6u}} \quad , \quad e^{v^c} = e^{L^c\,\tau_\infty} \quad \left(\frac{\rho_{10}\;\rho_{20}}{\rho_{30}\;\rho_{40}} \right)^{\frac{1}{6u}} \quad , \tag{14}$$

where we also define $v^c = \sum_{i=1}^7 v^i$ and $L^c = \sum_{i=1}^7 L^i$, needed below.

Thus, asymptotically as $t \to \infty$, the $(10+1)$ – dimensional universe effectively becomes $(3+1)$ – dimensional. Also, dimensional reduction of M theory along, for example, x^1 direction gives string theory with its dilaton now stabilised. Let the coordinate sizes $\simeq \mathcal{O}(\frac{1}{M_{11}})$. Then, upto numerical factors of $\mathcal{O}(1)$, the corresponding scales $(M_{11},\; M_4,\; M_s)$ and the string coupling constant g_s are related asymptotically by

$$M_4^2 \simeq \; e^{v^c}\; M_{11}^2 \simeq e^{v^c - v^1}\; M_s^2 \quad , \qquad g_s^2 \simeq e^{3 v^1} \quad . \tag{15}$$

5. To determine the sizes of brane directions and the relations in equation (15) explicitly for a given set of initial values $(l_0^I,\ K^I,\ L^i)$, we need τ_∞ if $L^i \neq 0$. We will obtain τ_∞ numerically since it depends on the details of evolution and we do not have explicit solutions. But we first give an approximate expression for τ_∞ which is easy to evaluate and works well under certain conditions.

Let $L^i \neq 0$. We set $E = 1$ by measuring t and τ in units of $\frac{1}{\sqrt{E}}$. Note that if $e^{l_0^I} \ll 1$ for all I then equations (8) and (9) imply that $l^I(\tau)$ may be taken as evolving 'freely', *i.e.* $l^I(\tau) = l_0^I + K^I \tau$ where $K^I = l_\tau^I(0) > 0$, until one of the $e^{l^I} = 1$; from then on, all e^{l^I} will evolve quickly and diverge soon after. Consequently, τ_∞ may be given approximately by

$$\tau_\infty \simeq \tau_a = min\ \{-\frac{l_0^I}{K^I}\} \ . \tag{16}$$

Also, τ_a is maximum, and $\tau_{a,\, max} = \frac{1}{K}$, when $K^1 = x^1$, $K^2 = x^2$, $K^3 = min\ \{x^1 + x^2,\ x^3\}$ and $K^4 = min\ \{x^1 + x^2,\ \frac{1}{2}\ (x^1 + x^2 + x^3),\ x^4\}$ where $x^I = -l_0^I K$, equation (9) at $\tau = 0$ determines $K > 0$, and we assume with no loss of generality that $0 < x^1 \le \cdots \le x^4$. No explicit solution is needed to evaluate τ_a and $\tau_{a,\, max}$.

We studied several sets of $(l_0^I,\ K^I)$ numerically and obtained $\tau_{\infty,\, max}$, the maximum of τ_∞ , by sampling 25000 random sets of K^I for each set of l_0^I . We find that l^I all diverge at finite $\tau = \tau_\infty$ and that, when $e^{l_0^I} \ll 1$ for all I , the approximations given above are quite good : $l^I > l_0^I + K^I \tau$ discernibly only for $\tau \gtrsim \tau_\infty - 4$; $\tau_a \sim (0.5 - 1.1)\ \tau_\infty$ generically; and, for K^I which maximise τ_a , we get $\tau_a = \tau_{a,\, max} \sim (0.9 - 1.1)\ \tau_\infty \sim (0.9 - 1.1)\ \tau_{\infty,\, max}$.

To convey an idea of what values are possible in equation (15), and also an idea of how good the approximations given above are, we consider two illustrative sets of l_0^I , choose K^I which maximise τ_a , choose $L^i = \sqrt{\frac{1}{6}}\ (-1, 2, 2, -1, 0, 0, 0, -1, -1, -1)$ so that g_s can be small, and choose $u = \frac{2}{3}$ which corresponds to radiation filled universe in $(3 + 1)$ – dimensions. The corresponding numerical results are given in Table I, from which e^{v^1} and e^{v^c} can be read off easily using equation (15). Also, $(\tau_{a,\, max},\ \tau_{\infty,\, max}) = (5.27,\ 5.82)$ for the first set, and $= (25.43,\ 25.69)$ for the second set of l_0^I in Table I.

$\{-l_0^I = -ln\ \rho_{I0}\}$	τ_∞	$\frac{M_{11}}{M_4}$	$\frac{M_s}{M_4}$	g_s
$5, 5, 12, 12$	5.73	$1.67 * 10^{-2}$	$2.17 * 10^{-3}$	$2.17 * 10^{-3}$
$20, 30, 40, 50$	25.64	$1.92 * 10^{-7}$	$1.97 * 10^{-12}$	$1.09 * 10^{-15}$

Table I : *The numerical results for* $(\tau_\infty;\ \frac{M_{11}}{M_4},\ \frac{M_s}{M_4},\ g_s)$ *for two illustrative sets of* l_0^I *. Other parameters are chosen as explained in the text.*

For a given set of l_0^I , our choice of $(K^I,\ L^i)$ in Table I results in near–minimum values for $(\frac{M_{11}}{M_4},\ \frac{M_s}{M_4},\ g_s)$ within about an order of magnitude. Our numerical studies confirm this. Also note that, since $E = 1$, $\lambda_t^i(0) = k^i \simeq K^I \simeq L^i \simeq \mathcal{O}(1)$ naturally whereas ensuring that $\rho_{I0} = e^{l_0^I} \ll 1$ for all I requires (fine) tuning. Thus, we conclude that our model naturally leads to $M_{11} \simeq M_s \simeq M_4$ and $g_s \simeq 1$ within a few orders of magnitude; and that smaller M_{11} and M_s , for example $M_s \simeq TeV \simeq 10^{-16}\ M_4$ as required in Large Volume compactification scenarios [3], are also possible but require a corresponding fine tuning of initial values.

6. We have shown that, in our model, three spatial directions expand and seven directions stabilise to constant sizes e^{v^i} , $i = 1, \cdots, 7$. We have also given exact expressions for v^i , which depend on initial values and τ_∞ . τ_∞ can be evaluated explicitly if solutions are known, otherwise numerically. Also, we give approximate expression for τ_∞ which is easy to evaluate and works well under certain conditions. Explicit relations among $(M_{11},\ M_4,\ M_s,\ g_s)$ then follow from which we see, for example, that obtaining $M_s \simeq TeV$ requires fine tuning.

We conclude by listing a few questions of obvious importance for further studies. **(i)** How to solve equations (8) – (11) analytically? **(ii)** Is there any way of obtaining $M_s \simeq TeV$ in the present model without fine tuning? **(iii)** Why $22'55'$ configuration and why not, for example, $22'2''$ (which will lead [9] to four spatial directions expanding)? The likely answer is that $22'55'$

configuration is entropically favourable [2, 8, 9], but dynamical details are not clear. **(iv)** What is the evolution when topology of spatial directions is more general? **(v)** We pointed out an interesting similarity with black holes. Does it have any deeper significance?

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